Positional Number Systems Decimal, Binary, Octal and Hexadecimal Numbers

Wakerly Section 2.1-2.3

Positional Number Systems

- The traditional number system is called a positional number system.
- A number is represented as a string of digits.
- Each digit position has a *weight* assoc. with it.
- Number's value = a *weighted sum* of the digits

$$6354 = 6*1000 + 3*100 + 5*10 + 4$$

$$D = \sum_{i=0}^{p-1} d_i 10^i$$

Fractions: Weights that are Negative Powers of 10

$$425.97 = 4*10^{2} + 2*10^{1} + 5*10^{0} + 9*10^{-1} + 7*10^{-2}$$

$$D = \sum_{i=-n}^{p-1} d_i 10^i$$

Binary Numbers

$$B = \sum_{i=-n}^{p-1} b_i 2^i$$

100101.0011

- The “base” is 2 instead of 10
- Meaning: the weights are powers of 2 instead of powers of 10.
- Digits are called “bits,” for “binary digits.”

Quiz

Convert the following binary numbers to decimal:

- 1011011.0110
- 00110.11001

Octal and Hexadecimal ("Hex") Numbers

- Octal = base 8
- Hexadecimal = base 16
  - Use A – F to represent the values 10 through 16 in each position.

Decimal	Binary	Octal	Hex
5	101	5	5
6	110	6	6
7	111	7	7
8	1000	10	8
9	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

Usefulness of Octal and Hex Numbers

- Useful for representing multibit binary numbers because their radices are integer multiples of 2.

$10\ 0101\ 1010\ 1111\ .\ 1011\ 111_2 = 2\ 5\ A\ F\ .\ B\ E_{16}$

Quiz: Convert from Binary to Octal:

- 1 101 011 110 111
- 11 011.101 1

Decimal-to-Radix-r Conversions

- Radix-r-to-decimal conversions are easy since we do arithmetic in decimal.
- However, decimal-to-radix-r conversions using decimal arithmetic is harder.
- To do the latter conversion, we convert the integer and fractional parts separately and add the results afterwards.

Decimal-to-Radix-r Conversions: Integer Part

- Successively divide number by r, taking remainder as result.
- Example: Convert 57_{10} to binary.

57 / 2 = 28 remainder 1 (LSB)

/2 = 14 remainder 0

/2 = 7 remainder 0

/2 = 3 remainder 1

/2 = 1 remainder 1

/2 = 0 remainder 1 (MSB)

Ans: 111001_2

Decimal-to-Radix-r Conversions: Fractional Part

- Successively multiply number by r, taking integer part as result and chopping off integer part before next iteration.
- May be unending!
- Example: convert $.3_{10}$ to binary.

.3 * 2 = .6 integer part = 0

.6 * 2 = 1.2 integer part = 1

.2 * 2 = .4 integer part = 0

.4 * 2 = .8 integer part = 0

.8 * 2 = 1.6 integer part = 1

.6 * 2 = 1.2 integer part = 1, etc.

Ans = $.0\dot{1}00\dot{1}$

Quiz

Convert from decimal to binary:

- 0.5
- 73.426
- 290.9